LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – APRIL 2010

MT 1500 - ALG.,ANAL.GEOMET. CAL. & TRIGN. - I

Date & Time: 28/04/2010 / 9:00 - 12:00 Dept. No. Max. : 100 Marks

PART - A

Answer ALL the questions **(10 × 2 = 20 marks)**

1) Write down the n^{th} derivative of $\frac{1}{(2x+3)^2}$.

2) Find the slope of the straight line $\frac{l}{r} = \cos(\theta - \alpha) + e\cos\theta$.

3) Write the formula for the radius of curvature when the pedal equation of the curve is given.

4) Define evolute of a curve.

5) If α, β, ν are the roots of the equation $x^3 + px^2 + qx + r = 0$ find the value of $\sum \alpha^2$.

6) Define a reciprocal equation.

7) Prove that $\cosh^{-1} x = \log(x + \sqrt{x^2 - 1})$.

8) Show that $1 - \tanh^2 x = \operatorname{sech}^2 x$.

9) Define pole and polar of a conic.

10) Define diameter and conjugate diameters of an ellipse.

PART - B

Answer any FIVE questions **(5 × 8 = 40 marks)**

11) Show that in the parabola $y^2 = 4ax$, the subtangent at any point is double the abscissa and the subnormal is constant.

12) Find the n^{th} derivative of $\sin^3 x \cos^2 x$.

13) Using Lagrange's multipliers find the maximum and minimum value of

f(x,y,z) = x+y+z subject to $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$.

14) Solve the equation $x^3 - 4x^2 - 3x + 18 = 0$ given that two of its roots are equal.

15) If α, β, ν are the roots of $x^3 + px^2 + qx + r = 0$ .prove that $(\alpha + \beta)(\beta + \nu)(\nu + \alpha) = r - pq$.

(P.T.O)

16) Expand $\cos 6\theta$ in terms of $\sin\theta$.

17) Show that the eccentric angles at the extremities of a pair of semi-conjugate diameters of an ellipse differ by a right angle .

18) Find the equations of the asymptotes and of the conjugate hyperbola given that the hyperbola has eccentricity $\sqrt{2}$, focus at the origin and the directrix along x +y + 1 = 0.

PART - C

Answer any TWO questions **($2\times 20 = 40$ marks)**

19. a) If y =acos(logx) + bsin(logx) then prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

b) Find the angle of intersection of the curves r = $\frac{a}{1+\cos\theta}$ and r = $\frac{b}{1-\cos\theta}$.

20. a) Find the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

b) Solve $6x^5+11x^4-33x^3-33x^2+11x+6=0$.

21. a) Calculate to two places of decimals the positive root of the equation $x^3+24x-50=0$ by Horner's method.

b) If $\tan(\alpha+i\beta)=x+iy$ prove that $x^2+y^2+2x\cot 2\alpha=1$.

22. a) Prove that $1-\frac{1}{2}\cos\theta+\frac{1\cdot 3}{2\cdot 4}\cos 2\theta-\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cos 3\theta+....\infty=\frac{\cos\frac{\theta}{4}}{\sqrt{2\cos\frac{\theta}{2}}}$.

b) A rectangular hyperbola whose centre is C is cut by any circle of radius r in four points P, Q, R, S. Prove that $CP^2+CQ^2+CR^2+CS^2=4r^2$.

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